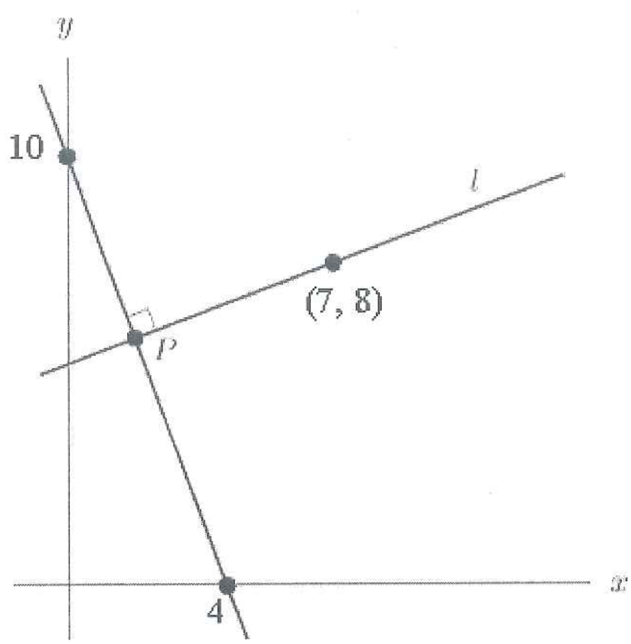
Cumulative Review Name: ______________________________

All answers must be justified with work. No work, no credit!

1. Find the slope of the line l shown n the following figure.



SLOPE OF OTHER LINE THROUGH (4,0)(0,10)

$M = \frac{\Delta y}{\Delta x} = \frac{10-0}{0-4} = \frac{10}{-4} = -\frac{5}{2}$

SLOPE OF l is negative reciprocal

$\boxed{M = \frac{2}{5}}$

2. The following chart gives the number of students in a class that are a specific height in inches

height	55 inches	60 inches	65 inches	70 inches	75 inches
number of students	5	6	6	1	0

$60 \to 6$
$65 \to 6$ OKAY

a) Is the number of students in each category a function of the height? YES, EACH X HAS EXACTLY ONE Y-VALUE

b) Is the height in each category a function of the number of students in that category?

NO, AN X-VALUE HAS 2 Y-VALUES

$6 \to 60$, $\to 65$ NOT OKAY

3. Are the lines $y = -0.5x + 1$ and $y = 0.5x - 5$ parallel, perpendicular, or neither?

SLOPE $= -\frac{1}{2}$ SLOPE $= \frac{1}{2}$ NEITHER

NOT THE SAME (PARALLEL)
NOT NEGATIVE RECIPROCALS (PERP)

4. Which of the following are *not* in the domain of $r(x) = \frac{1}{(x+7)^2} + \sqrt{1-x}$?

MUST BE POSITIVE

A) $x = 7$
B) $x = -7$
C) $x = 1$
D) All numbers x such that $x > 1$
E) All numbers x such that $x < 1$

$(x+7)^2 \neq 0$
$x+7 \neq 0$
$x \neq -7$

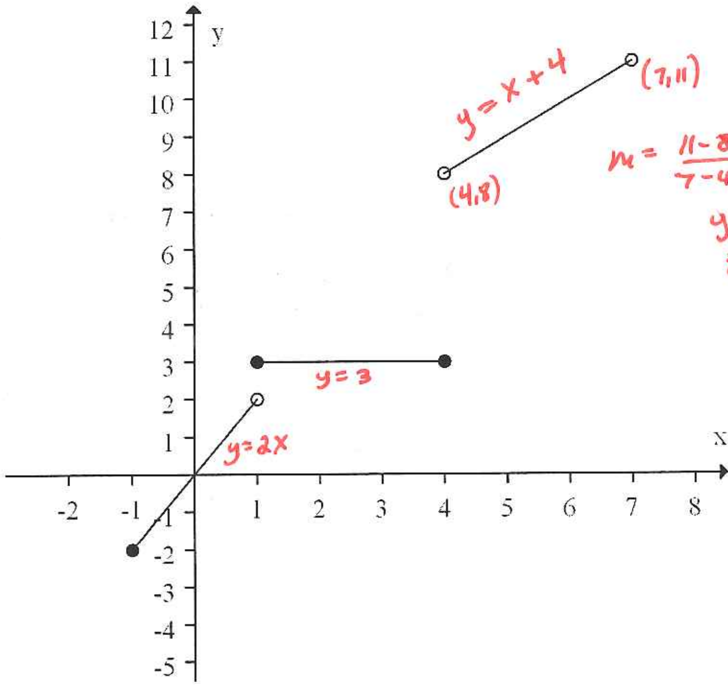
$1 - x \geq 0$
$-x \geq -1$
$x \leq 1$

ARE IN THE DOMAIN

$x > 1 \leftarrow$ ARE NOT

5. Find a formula for the following graph:

$$y = \begin{cases} 2x & \text{if } -1 \le x < 1 \\ 3 & \text{if } 1 \le x \le 4 \\ x+4 & \text{if } 4 < x < 7 \end{cases}$$



$m = \frac{11-8}{7-4} = \frac{3}{3} = 1$

$y = mx + b$

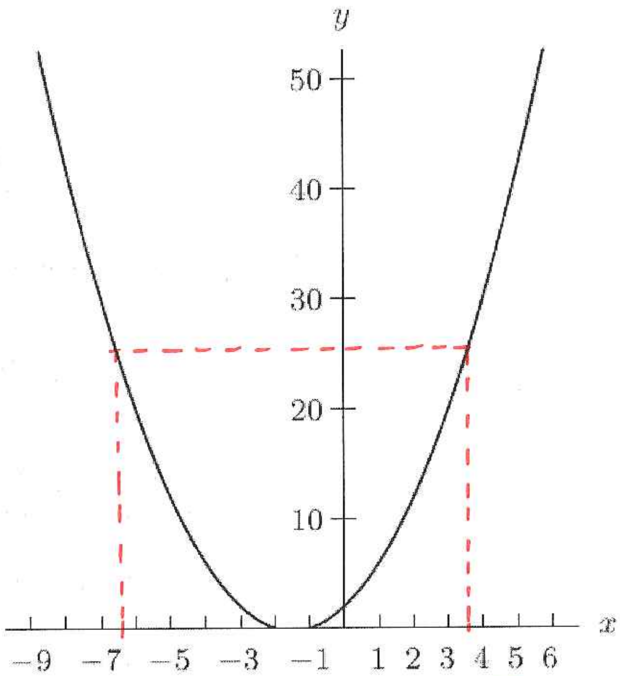
$8 = 1(4) + b$

$8 = 4 + b$

$4 = b$

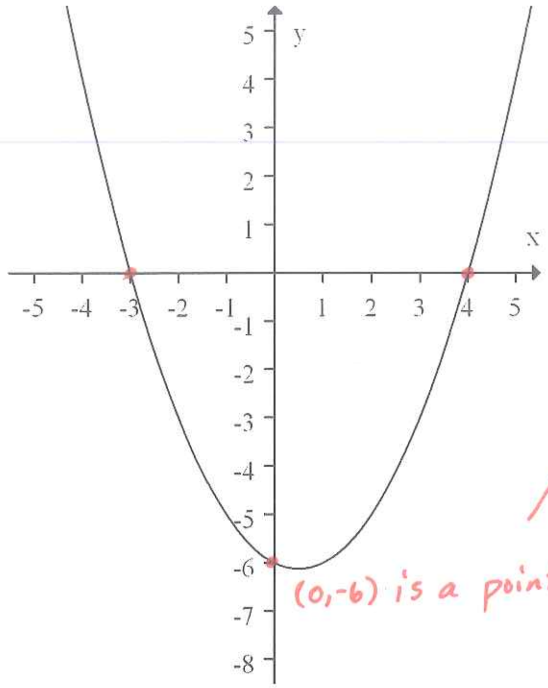
6. Use the graph of $f(x) = x^2 + 3x + 2$ given below to estimate $f^{-1}(25)$. (Mark all correct answers)

↑ find x when y is 25



A) 3.5 B) – 3.5 C) 6.5 D) – 6.5 E) – 1.5 F) 1.5

7. Find a formula for the parabola



(0,-6) is a point

ZEROS: 4 , -3

FACTORS: $(x-4)(x+3)$

$y = a(x-4)(x+3)$

$-6 = a(0-4)(0+3)$

$-6 = a(-4)(3)$

$-6 = -12a$

$\frac{1}{2} = a$

$\boxed{y = \frac{1}{2}(x-4)(x+3)}$

8. Given $f(x) = 4x^2 - 2$ and $g(x) = -3x + 1$, find $f(g(4))$ and $g(f(4))$.

$g(4) = -3(4) + 1$

$= -12 + 1$

$g(4) = -11$

$f(g(4)) = f(-11) = 4(-11)^2 - 2$

$= 4(121) - 2$

$= 484 - 2$

$\boxed{f(g(4)) = 482}$

$f(4) = 4(4)^2 - 2$

$= 4(16) - 2$

$= 64 - 2$

$= 62$

$g(f(4)) = g(62) = -3(62) + 1$

$= -186 + 1$

$\boxed{g(f(4)) = -185}$

9. What is the equation of the parabola that is concave down, has vertex $(-1, 5)$ and contains the origin.

VERTEX FORM: $y = a(x-h)^2 + k$

$y = a(x+1)^2 + 5$

$0 = a(0+1)^2 + 5$ ← Substitute (0,0) for x and y

$0 = a + 5$

$-5 = a$

$\boxed{y = -5(x+1)^2 + 5}$

10. Complete the square in order to write the following function in vertex form: $f(x) = x^2 + 2x - 10$.

$y = x^2 + 2x \quad -10$

$= x^2 + 2x + 1 \quad -1 - 10$ BALANCE

$\left(\frac{b}{2}\right)^2$

$\boxed{y = (x+1)^2 - 11}$

11. The formula for the exponential function P such that $P(3) = 24$ and $P(9) = 6$ is given by $P(t) =$ $\underline{48.00}$ ($\underline{.79}$)^t. Give both answers to 2 decimal places.

$y = ab^x$

$24 = ab^3$

$6 = ab^9$

$4 = b^{-6}$

$4 = \frac{1}{b^6}$

$4b^6 = 1$

$b^6 = \frac{1}{4}$

$b = \left(\frac{1}{4}\right)^{\frac{1}{6}}$

$b = .7937$

$y = ab^x$

$24 = a(.7937)^3$

$\frac{24}{.7937^3} = a$

$48 = a$

12. Write a formula that gives the value in an account after t years. Assume that the initial value in the account is \$1500 and that the account doubles in value every 10 years.

$y = ab^t$

$y = 1500 \cdot 2^{\frac{t}{10}}$

13. Kathleen opens a savings account with \$1300. The account earns 3.2% annual interest compounded quarterly. How much will be in the account after 13 years?

$y = 1300\left(1 + \frac{.032}{4}\right)^{13 \cdot 4}$

$\boxed{y = \$1967.39}$

14. Let $P(t) = 500e^{0.05t}$ give the size of a population of animals in year t. After how many years will the population be approximately 1007?

$1007 = 500e^{.05t}$

$\frac{1007}{500} = e^{.05t}$

$\ln\left(\frac{1007}{500}\right) = \ln\left(e^{.05t}\right)$

$\frac{\ln\left(\frac{1007}{500}\right)}{.05} = \frac{.05t}{.05}$

$14.002 = t$

$\boxed{\text{Approximately 14 years}}$

15. Rewriting $e^{3a} = b$ using logs gives

A) $\ln a = b/3$

B) $\ln(b/3) = a$

C) $\ln 3a = b$

D) $\ln b = 3a$

$\log_e b = 3a$

$\ln b = 3a$

16. Let $n = \log p$ and $m = \log q$. What is $\log \frac{p^3}{q^6}$?

A) $\frac{n^3}{m^6}$

B) $\left(\frac{n}{m}\right)^{-3}$

C) $(n-m)^{-3}$

D) $3n-6m$

$\log\left(\frac{p^3}{q^6}\right) = \log(p^3) - \log(q^6)$
$= 3\log p - 6\log q$
$= 3n - 6m$

17. The doubling time for a bank account that is growing by 5.1% per year (compounded continually) is 13.6 years. Round 1 decimal place.

$A = A_0 e^{.051t}$
$2A_0 = A_0 e^{.051t}$
$2 = e^{.051t}$
$\ln 2 = \ln(e^{.051t})$
$\frac{\ln 2}{.051} = \frac{.051t}{.051}$

$t = 13.591$
$t = 13.6$ years

18. What is the effect of the translation $f(x-6a)+b$ on the graph of the function $f(x)$? Assume a and b are positive constants.

right 6a up b

A) Shift right 6a, then up b.
B) Shift right 6a, then down b.
C) Shift left 6a, then up b.
D) Shift left 6a, then down b.

19. The graph of a function f has been shifted down 4 units, shifted 5 units to the right, and then stretched vertically by a factor of 8. The new graph is produced by a function g. Find a formula for g in terms of f.

$g(x) = f(x) - 4$
$= f(x-5) - 4$
$= 8(f(x-5) - 4)$
$g(x) = 8f(x-5) - 32$

20. Find the coordinates of the point on the unit circle with angle α if $\cos\alpha = 0.630$. Round each coordinate to 3 decimal places.

ANY COORDINATE ON A CIRCLE $(r\cos\theta, r\sin\theta) = (.630, \sin(\cos^{-1}(.630)))$

$\alpha = \cos^{-1}(.630)$

$(.630, .777)$

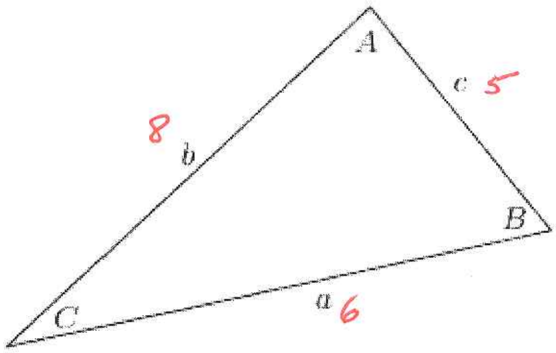
21. Find the coordinates of the point at angle -23° on a circle of radius 7.1. Round each coordinate to 3 decimal places, if necessary.

$(r\cos\theta, r\sin\theta)$

$(7.1\cos(-23^{\circ}), 7.1\sin(-23^{\circ}))$

$(6.536, -2.774)$

22. Find the measures of the angles of the triangle if $a = 6$, $b = 8$, and $c = 5$. Round to two decimal places.



LARGEST ANGLE FIRST

$b^2 = a^2 + c^2 - 2ac\cos B$

$8^2 = 6^2 + 5^2 - 2(6)(5)\cos B$

$64 = 61 - 60\cos B$

$3 = -60\cos B$

$-\frac{3}{60} = \cos B$

$B = \cos^{-1}(-\frac{3}{60})$

$\boxed{B = 92.87^{\circ}}$

$\frac{\sin 92.86598}{8} = \frac{\sin A}{6}$

$\frac{8\sin A}{8} = \frac{6(\sin 92.86598)}{8}$

$\sin A = \frac{6\sin 92.86598}{8}$

$A = \sin^{-1}\left(\frac{6\sin 92.86598}{8}\right)$

$\boxed{A = 48.51^{\circ}}$

$C = 180 - 92.87 - 48.51$

$\boxed{C = 38.62^{\circ}}$

23. What is the amplitude, period, and midline of the periodic function $y = 2\cos(2x) - 8$?

Amp ↑ B ↑ Midline ↑

$P = \frac{2\pi}{B}$

$= \frac{2\pi}{2}$

$P = \pi$

$\boxed{\begin{array}{l}\text{AMPLITUDE} = 2 \\ \text{PERIOD} = \pi \\ \text{MIDLINE: } y = -8\end{array}}$

24. The angle 135° is equivalent to $\underline{\frac{3}{4}}$ π radians.

$\frac{135}{180} = \frac{\theta}{\pi}$

$\frac{180\theta}{180} = \frac{135\pi}{180}$

$\theta = \frac{27\pi}{36}$

$\theta = \frac{3\pi}{4}$

25. What is the length of an arc cut off by an angle of 210° in a circle of radius 2.9 meters? Give your answer correct to 3 decimal places.

$\frac{210^{\circ}}{360^{\circ}} = \frac{S}{2\pi(2.9)}$

$\frac{360S}{360} = \frac{210(2\pi)(2.9)}{360}$

$S = 10.629$ meters

26. For the following function, identify the amplitude, period, horizontal shift, and vertical shift and then graph it from $-8\pi \le t \le 8\pi$:

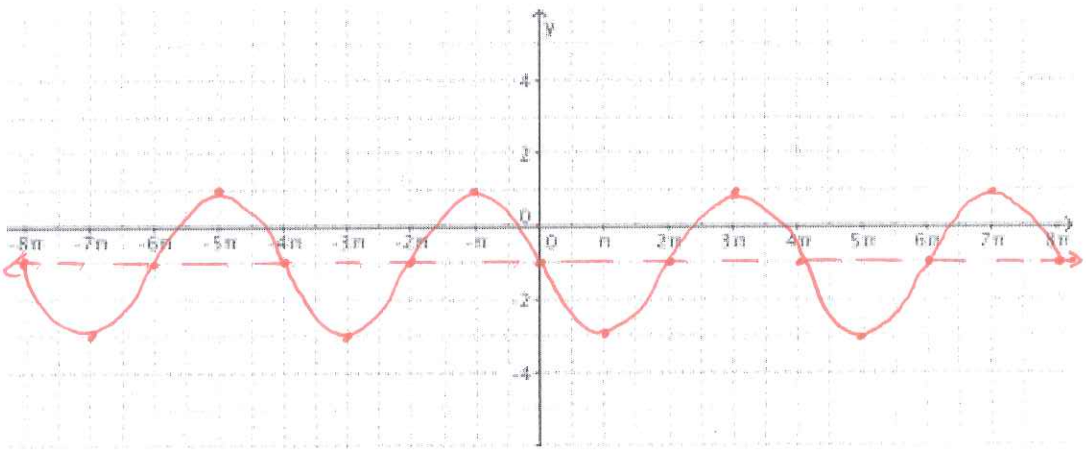
$$f(t) = 2\sin\left(\frac{1}{2}t - \pi\right) - 1$$

Amp: 2

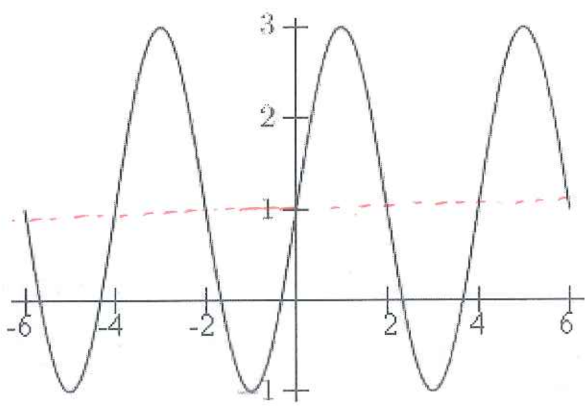
Period $= \frac{2\pi}{B} = \frac{2\pi}{\frac{1}{2}} = 4\pi$

Hor SHIFT $= \frac{-c}{a} = \frac{\pi}{\frac{1}{2}} = 2\pi$

VERT SHIFT: -1



27. The formula for the following trigonometric function is $f(t) = \underline{\ 2\ }\sin(\underline{\ \frac{1}{2}\ }\pi t) + \underline{\ 1\ }$.



Amp = 2

Period = 4

Midline: $y = 1$

$P = \frac{2\pi}{B}$

$\frac{4}{1} = \frac{2\pi}{B}$

$4B = 2\pi$

$B = \frac{2\pi}{4} = \frac{\pi}{2}$

28. Find a formula for a deer population which oscillates over a 6 year period between a low of 1000 in year $t = 0$ and a high of 3500 in year $t = 3$.

Reflected Cosine - starts down

midline: $y = \frac{1000 + 3500}{2}$

$y = 2250$

Amp $= \frac{3500 - 1000}{2}$

Amp $= 1250$

$P = \frac{2\pi}{B}$

$\frac{6}{1} = \frac{2\pi}{B}$

$6B = 2\pi$

$B = \frac{2\pi}{6} = \frac{\pi}{3}$

$\boxed{y = -1250\cos\left(\frac{\pi}{3}t\right) + 2250}$

Reflection

29. A ferris wheel sitting on the ground is 24 meters in diameter and makes one revolution every 7 minutes. If you start in the 9 o'clock position $t=0$ and the wheel is rotating clockwise, write a formula for your height above the ground at time t.

Midline: $y=12$ Period = 7 min Amplitude: $\frac{24}{2}=12$

$P=\frac{2\pi}{B}$ $B=\frac{2\pi}{7}$

$\frac{7}{1}=\frac{2\pi}{B}$

$7B=2\pi$

$\boxed{y=12\sin\left(\frac{2\pi}{7}t\right)+12}$

30. For positive numbers x, what is the inverse of $h(x)=e^{\sqrt{x}-7}$?

$y=e^{\sqrt{x}-7}$

$x=e^{\sqrt{y}-7}$

$\ln x=\ln\left(e^{\sqrt{y}-7}\right)$

$\ln x=\sqrt{y}-7$

$\ln x+7=\sqrt{y}$

$\boxed{(\ln x+7)^2=y}$

$h^{-1}(x)=(\ln x+7)^2$

31. Let $g(x)=\frac{3}{x}+6$. Use composition of functions to check/prove that $g^{-1}(x)=\frac{3}{x-6}$.

$g(g^{-1}(x))=\frac{3}{\frac{3}{x-6}}+6$

$=3\cdot\frac{x-6}{3}+6$

$=x-6+6$

$=x$ ✓

$g^{-1}(g(x))=\frac{3}{\frac{3}{x}+6-6}$

$=\frac{3}{\frac{3}{x}}$

$=3\cdot\frac{x}{3}$

$=x$ ✓

32. Given $f^{-1}(x)=300(1.03)^x$, solve $f^{-1}(x)=350$. Round to 3 decimal places.

$350=300(1.03)^x$

$\frac{350}{300}=1.03^x$

$\log\left(\frac{7}{6}\right)=\log\left(1.03^x\right)$

$\frac{\log\left(\frac{7}{6}\right)}{\log 1.03}=\frac{x\log 1.03}{\log 1.03}$

$\boxed{5.215=x}$

33. The power function through the point (2, 5) and (8, 12) is $y=kx^p$, where $k=$ <u>3.227</u> and $p=$ <u>.632</u>. Round the second answer to 3 decimal places.

$y=kx^p$

$\frac{12=k(8)^p}{5=k(2)^p}$

$\frac{12}{5}=4^p$

$\log\left(\frac{12}{5}\right)=\log 4^p$

$\frac{\log\left(\frac{12}{5}\right)}{\log 4}=\frac{p\log 4}{\log 4}$

$.6315172=p$

$.632=p$

$y=kx^p$

$\frac{5=k(2)^{.6315172}}{2^{.6315172}\quad 2^{.6315172}}$

$3.22749=k$

$3.227=k$

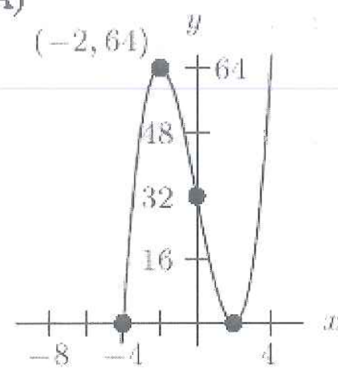
ZEROS: $X = 2 \quad X = -4$

34. Let $f(x) = (x-2)^2(x+4)$. Which of the following figures shows the graph of $f(-0.5x)$?

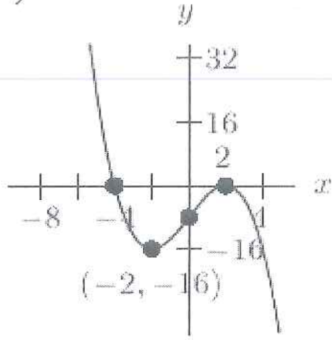
HORIZONTAL STRETCH SF 2

HORIZONTAL REFLECTION

(A)



(B)



Reflect!

$X = 2 \rightarrow X = -2$

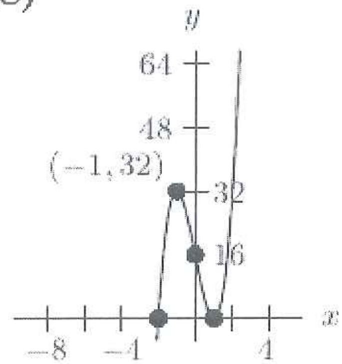
$X = -4 \rightarrow X = 4$

STRETCH SF 2

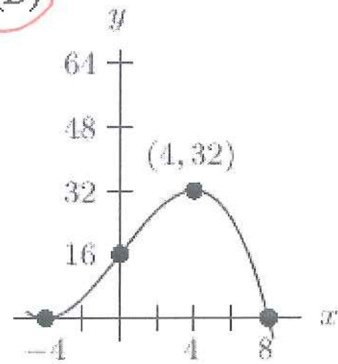
$X = -2 \rightarrow X = -4$

$X = 4 \rightarrow X = 8$

(C)



(D)



35. Which of the following are polynomials:

A) $y = 15x^4$

B) $y = (x^2+2)(x-15)e^x$ EXPONENTIAL

C) $y = 1 - 15t + \sqrt{2t}^5$ FRACTIONAL EXPONENT

D) $y = 1 - 15t + \sqrt{2t^5}$

36. Compute the following limits:

a) $\lim_{x\to\infty}(-5x^4 + 7x^3 - 116x^2)$ $\quad \lim_{X\to\infty} -5X^4 = -\infty$

b) $\lim_{x\to-\infty}(-5x^4 + 7x^3 - 116x^2)$ $\quad \lim_{X\to-\infty} -5X^4 = -\infty$

37. Find all intercepts, zeros, asymptotes, and holes of: $f(x) = \dfrac{x^2 - 25}{x^2 + 6x}$?

$f(x) = \dfrac{(x+5)(x-5)}{x(x+6)}$

Zeros $x+5=0$ $x-5=0$ $\boxed{x=-5 \quad x=5}$

VA: $\boxed{x=0}$ $x+6=0$ $\boxed{x=-6}$

$f(0) = \dfrac{(0+5)(0-5)}{0(0+6)}$

undefined, no y-intercept

HA: $\dfrac{x^2}{x^2} = \boxed{y=1}$

No HOLES

38. As $x \to +\infty$, $f(x) = -4x^3 + 6x^2 + 17 \to$ $\underline{-\infty}$. Enter "infinity " or "- infinity" for ∞ or $-\infty$.

as $x \to \infty$ $-4x^3 \to -\infty$